Berechnung der Koeffizienten von Ausgleichsproblemen

Dominik Zobel, 2011

Ausgleichsproblem der Form $y = be^{c\mathbf{x}}$:

$$\ln(b) = \frac{\sum_{i=1}^{n}(\mathbf{x}_i)\sum_{i=1}^{n}(\mathbf{x}_i\ln(\mathbf{y}_i)) - \sum_{i=1}^{n}(\mathbf{x}_i^2)\sum_{i=1}^{n}(\ln(\mathbf{y}_i))}{\sum_{i=1}^{n}(\mathbf{x}_i)\sum_{i=1}^{n}(\mathbf{x}_i) - n\sum_{i=1}^{n}(\mathbf{x}_i^2)}$$

$$c = \frac{\sum_{i=1}^{n}(\mathbf{x}_i\ln(\mathbf{y}_i)) - \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i)\sum_{i=1}^{n}(\ln(\mathbf{y}_i))}{\sum_{i=1}^{n}(\mathbf{x}_i^2) - \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i)\sum_{i=1}^{n}(\mathbf{x}_i)}$$

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b=exp((sum(x)*(x*log(y)')-x*x'*sum(log(y)))/(sum(x)*sum(x)-size(y,2)*x*x'));
c=-(x*log(y)'-1/size(y,2)*sum(x)*sum(log(y)))/(x*x'-1/size(y,2)*sum(x)*sum(x));
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Ausgleichsproblem der Form $y = a\mathbf{x}^{\alpha}$:

$$a = \frac{\sum_{i=1}^{n}(\mathbf{x}_i^{\alpha}\mathbf{y}_i)}{\sum_{i=1}^{n}(\mathbf{x}_i^{2\alpha})}$$

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a=(x.^alpha*y')/(x.^alpha*(x.^alpha)');
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Ausgleichsproblem der Form $y = a\mathbf{x}^{\alpha} + b\mathbf{x}^{\beta}$:

$$a = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\sum_{i=1}^{n}(\mathbf{x}_i^{\alpha}\mathbf{y}_i) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)}{\sum_{i=1}^{n}(\mathbf{x}_i^{2\alpha})\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right) - \left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\right)^2}$$

$$b = \frac{\sum_{i=1}^{n}(\mathbf{x}_i^{2\alpha})\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}(\mathbf{x}_i^{\alpha}\mathbf{y}_i)}{\sum_{i=1}^{n}(\mathbf{x}_i^{2\alpha})\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right) - \left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\right)^2}$$

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a=(x.^beta*(x.^beta)'*x.^alpha*y'-x.^alpha*(x.^beta)'*x.^beta*y')/(x.^alpha*(x.^alpha)'*x.^beta*(x.^beta)'-(x.^alpha*(x.^beta)')^2);
b=(x.^alpha*(x.^alpha)'*x.^beta*y'-x.^alpha*(x.^beta)'*x.^alpha*y')/(x.^alpha*(x.^alpha)'*x.^beta*(x.^beta)'-(x.^alpha*(x.^beta)')^2);
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Auf Grund der Symmetrie kann die Formel zur Berechnung von a auch für b benutzt werden. Dazu müssen lediglich α und β in der Formel getauscht werden.

Ausgleichsproblem der Form $y = a\mathbf{x}^{\alpha} + b\mathbf{x}^{\beta} + c\mathbf{x}^{\gamma}$:

$$a = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha}\mathbf{y}_i\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right) + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)}{2\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right)}$$

$$b = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right) + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)}{2\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right)}$$

$$c = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma}\mathbf{y}_i\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\alpha}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma+\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\alpha}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right) + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\alpha}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\gamma+\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)}{2\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\beta+\gamma}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\gamma}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha+\beta}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\beta}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\gamma}\right)}$$

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a=(x.^alpha*y'*x.^beta*(x.^beta)'*x.^gamma*(x.^gamma)'-x.^alpha*y'*(x.^beta*(x.^gamma)')^2+x.^beta*y'*x.^alpha*(x.^gamma)'*x.^beta*(x.^gamma)'-x.^beta*y'*x.^alpha*(x.^beta)'*x.^gamma*(x.^gamma)' ...
  +x.^gamma*y'*x.^alpha*(x.^beta)'*x.^beta*(x.^gamma)'-x.^gamma*y'*x.^alpha*(x.^gamma)'*x.^beta*(x.^beta)')/(x.^alpha*(x.^beta)'*x.^alpha*(x.^gamma)'*x.^beta*(x.^gamma)'*2 ...
  -x.^alpha*(x.^alpha)'*(x.^beta*(x.^gamma)')^2-x.^beta*(x.^beta)'*(x.^alpha*(x.^gamma)')^2-x.^gamma*(x.^gamma)'*(x.^alpha*(x.^beta)')^2+x.^alpha*(x.^alpha)'*x.^beta*(x.^beta)'*x.^gamma*(x.^gamma)');
b=(x.^beta*y'*x.^alpha*(x.^alpha)'*x.^gamma*(x.^gamma)'-x.^beta*y'*(x.^alpha*(x.^gamma)')^2+x.^alpha*y'*x.^alpha*(x.^gamma)'*x.^beta*(x.^gamma)'-x.^alpha*y'*x.^alpha*(x.^beta)'*x.^gamma*(x.^gamma)' ...
  +x.^gamma*y'*x.^alpha*(x.^beta)'*x.^alpha*(x.^gamma)'-x.^gamma*y'*x.^alpha*(x.^alpha)'*x.^beta*(x.^gamma)')/(x.^alpha*(x.^beta)'*x.^alpha*(x.^gamma)'*x.^beta*(x.^gamma)'*2 ...
  -x.^alpha*(x.^alpha)'*(x.^beta*(x.^gamma)')^2-x.^beta*(x.^beta)'*(x.^alpha*(x.^gamma)')^2-x.^gamma*(x.^gamma)'*(x.^alpha*(x.^beta)')^2+x.^alpha*(x.^alpha)'*x.^beta*(x.^beta)'*x.^gamma*(x.^gamma)');
c=(x.^gamma*y'*x.^alpha*(x.^alpha)'*x.^beta*(x.^beta)'-x.^gamma*y'*(x.^alpha*(x.^beta)')^2+x.^beta*y'*x.^alpha*(x.^beta)'*x.^alpha*(x.^gamma)'-x.^beta*y'*x.^alpha*(x.^alpha)'*x.^beta*(x.^gamma)' ...
  +x.^alpha*y'*x.^alpha*(x.^beta)'*x.^beta*(x.^gamma)'-x.^alpha*y'*x.^beta*(x.^beta)'*x.^alpha*(x.^gamma)')/(x.^alpha*(x.^beta)'*x.^alpha*(x.^gamma)'*x.^beta*(x.^gamma)'*2 ...
  -x.^alpha*(x.^alpha)'*(x.^beta*(x.^gamma)')^2-x.^beta*(x.^beta)'*(x.^alpha*(x.^gamma)')^2-x.^gamma*(x.^gamma)'*(x.^alpha*(x.^beta)')^2+x.^alpha*(x.^alpha)'*x.^beta*(x.^beta)'*x.^gamma*(x.^gamma)');
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Auf Grund der Symmetrie kann die Formel zur Berechnung von a auch für b und c benutzt werden. Dazu müssen lediglich α und β bzw. γ getauscht werden.

Die ersten drei rekursiven Formeln (bis zum Grad t)

$$k_1 = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{y}_i\right)}{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)} - \sum_{r=2}^{t}\left(k_r\frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{x}_i^{\alpha_r}\right)}{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)}\right)$$

$$k_2 = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{y}_i\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2}\mathbf{y}_i\right)}{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right) - \left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\right)^2} - \sum_{r=3}^{t}\left(k_r\frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{x}_i^{\alpha_r}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2}\mathbf{x}_i^{\alpha_r}\right)}{\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right) - \left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\right)^2}\right)$$

$$k_3 = \frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{y}_i\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right) + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_3}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_3}\mathbf{y}_i\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)}{2\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right)}$$
$$- \sum_{r=4}^{t}\left(k_r\frac{\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{x}_i^{\alpha_r}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1}\mathbf{x}_i^{\alpha_r}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2}\mathbf{x}_i^{\alpha_r}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2}\mathbf{x}_i^{\alpha_r}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right) + \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_3}\mathbf{x}_i^{\alpha_r}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_3}\mathbf{x}_i^{\alpha_r}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)}{2\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right) - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_2+\alpha_3}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_3}\right)\right)^2 - \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right)\left(\sum_{i=1}^{n}\left(\mathbf{x}_i^{\alpha_1+\alpha_2}\right)\right)^2 + \sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_1}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_2}\right)\sum_{i=1}^{n}\left(\mathbf{x}_i^{2\alpha_3}\right)}\right)$$